The domains of convergence of the degree series of variable of the complex numbers

Khaitova Khilola Gafurovna
teacher of the Department of Mathematical Analysis, Bukhara State University,
Mail: x.xayitova@mail.ru

Abstract. This research considers two traditional important questions, which are interesting, at least to most mathematicians. The first question arises in the theory of the degree series of variable of the complex numbers, which concerns the relationship, if any, and their the domains of convergence. The second question arises in the theory of the degree series of several variables of the complex numbers. In this research we introduce double series and we give the definition of their convergence and divergence.

Key words: double degree series, convergence, divergence.

We discuss the geometrical structure of following double degree series

$$\sum_{n+m=0}^{\infty} a_{nm}x^n y^m \qquad (1)$$

Abel's theorem. If double degree series has absolute converges in point (x_0, y_0) $x_0 \neq 0, y_0 \neq 0,$
then, in direct rectangle
$T = \{(x,y): |x| < |x_0|, |y| < |y_0|\}$
it will have absolute and uniformly converges.

Abel theorem needs convergence of the degree series of several variables for $x_0 \neq 0$ points. In other words its enough that point has conditional convergence. But not for the double degree series.

We have some examples of the degree series which hasn't any converges except of $(1,1)$ and $(0,0)$ points. (look at $[1]$)

It we need the convergence at line in $(x_0, 0)$ or $(0, y_0)$ points, we'll have the convergence of (1) line in $|x| < |x_0|$ or $|y| < |y_0|$ on direct line.

Counting on Abel theorem we'll have following theorem:

Theorem. The domain of convergence of the double degree series will be symmetric to the OX and OY axle.

Following examples show the variety of convergence of the double series.

For example,

$$\sum_{n=0}^{\infty} (x^2 + y^2)^n$$

the domain of convergence of this degree series consist of
$B = \{(a, y): \ |x^2 + y^2| < 1\}.$
And the domain of convergence of this degree series:

$$\sum_{n=0}^{\infty} (x^2 - y^2)^n$$

consists of

$G = \{(x, y): \ |x^2 - y^2| < 1\}.$

This domain of convergence

$x^2 - y^2 = 1$ and $y^2 - x^2 < 1$

limited by hyperboloid.

$\sum_{n=0}^{\infty} (x - y)^n$

degree series has converges in

$P = \{(x, y): \ |x - y| < 1\}$

and limited by

$x - y = 1$ and $y - x = 1.$

All symmetric domains to OX or OY axles have converges with some degree series, in other words there is always some degree series which has converges only in requested domain and has no converges out of this domain.

The purpose of this dissertation is to define and study a class of convergent double series and which will be of use in accelerating their convergence.

In this research we describe some of the simplest examples of domains in the space C^n. As usual, a domain is an open connected set, where openness means that along with any point of it the set also contains a neighborhood of that point, and connectedness of an open set D means that, for any points

$'z, ''z \epsilon D$

there exists a continuous are

$\gamma: [0,1] \to D.$

In this research we describe some domains of double degree series:

-the ball of radius r with center at the point $a \epsilon C^n$ is defined as the set of points

$B(a, r) = \{z \epsilon C^n: \ |z - a| < r\};$

-the polydisc of radius with center $a \epsilon C^n$ is defined as the set of points

$u(a, r) = \{z \epsilon C^n: \ ||z - a|| < r\};$

-Reinhardt domains with center at a point $a \epsilon C^n$ are defined as domains that possess the following property: for any point

$z^0 = \{z_v^0\}$

that belongs to the domain, the domain also contains any point

$z = \{a_v + (z_v^0 - a_v)e^{iv}\}, \ 0 < a_v < 2\pi;$

-Hartogs domains with symmetry plane

$z^0 = \{z_v^0, \ v = 1,2, \dots, n-1\}$ and $|z_n - a_n| \leq |z_n^0 - a_n|;$

-circular domains with center at a point $a \epsilon C^n$ are domains which, for each point z, also contain all the points

$a + (z - a)e^{i\rho}, \ 0 < \rho < 2\pi.$

REFERENCES

1. B.V. Shabat, Introduction to Complex Analysis – excerpts, Part 1, Part 2, 2003. P.p 1-3 chapter, 1-chapter.
2. George Cain, Complex Analysis, 1999 , P.p 8-11 chapter.